



The World Rides On Us®

Experience with Strain Energy Based Strain-Life Property Prediction

James J. Patterson, PhD.

10-28-20

CONFIDENTIAL

ϵ -N Curve Prediction (Hyst. Loops): Background...

- Dr. Peter Huffman, an SAE FD&E member, has developed a method to predict ϵ -N fatigue life curves.
- The only inputs needed are from a single fatigue test sample at a “high” strain rate...
 - E – Modulus of Elasticity
 - K' and n' - Ramburg-Osgood fit of the hysteresis loop
 - ϵ and $2N_f$ - Coordinate from the test
- If proven to be viable, this method could significantly reduce the time and cost associated with developing this data.

ϵ -N Curve Prediction (Hyst. Loops): Background...

- Data sets were provided to Dr. Huffman, but they were from the final fit of the full test regimen.
- A more likely, and favorable, scenario would be to supply a hysteresis loop from a single test...
 - Choose a stable loop midway (???) through the test.
 - Use a test with a reasonable strain level...
 - ◆ Large or small hysteresis loop?
- Derive E , K' , and n' from this data.
- Still need to supply the value for $2N_f$ at this strain level.

ε-N Curve Prediction (Hyst. Loops): Background...

DETERMINATION OF STRAIN-LIFE FATIGUE **PROPERTIES**

- When fitting the data to obtain the four strain-life properties, stress or plastic strain amplitude should be treated as independent variables, whereas the fatigue life is the dependent variable (i.e. fatigue life cannot be controlled and is dependent upon the applied strain amplitude).
- The cyclic strength coefficient, K' , and cyclic strain hardening exponent, n' , are obtained from fitting stable stress amplitude versus plastic strain amplitude data. Rough estimates of K' and n' can also be calculated from the low cycle fatigue properties by using:

$$(\sigma'_f / \epsilon'_f)^{b/c} / K' = 1$$

$$K' = \frac{\sigma'_f}{(\epsilon'_f)^{b/c}}$$

$$n' = \frac{b}{c}$$

$$(b/c) / n' = 1$$

These equations are derived from compatibility between strain-life equations.



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TOLEDO

Ali Fatemi-University of Toledo All Rights Reserved Chapter 5-Cyclic Deformation & ε-N Approach

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ϵ -N Curve Prediction (Hyst. Loops): Background...

- Compare the curves as a % difference from the final known curve.
- Compare the parameters between the test and the prediction.
- These would obviously not be possible in an actual situation.

***Strain-Life Curve Prediction
from Hysteresis Loops
MAR-5589a – Gr 50 Plate***

ε-N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-5589a – Gr 50...



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VEHICLE SYSTEMS

To _____
From _____ File.
Subject _____ Date

This report to cover one subject only

TEST PROGRAM SUMMARY

– Turn on formatting marks (¶) to show help text –

Cyclic Stress-Strain	$\frac{\Delta \epsilon}{2} = \frac{\Delta \sigma}{2E'} + \left(\frac{\Delta \sigma}{2K'} \right)^{n'}$	Cyclic Strain- Life	Cyclic Stress- Life	Stress Ratio, $R' = 12$ Strain Ratio, $n' = 0.120$ Fatigue Coefficient, $C' = 29.7E6$ psi
Elastic Strain-Life	$\frac{\Delta \epsilon_e}{2} = \frac{\sigma_f'}{E'} (2N_f)^b$	Fatigue Life	Fatigue Life	Fatigue Coefficient, $C' = 554$ psi
Plastic Strain-Life	$\frac{\Delta \epsilon_p}{2} = \epsilon_f' (2N_f)^c$	Fatigue Life	Fatigue Life	Fatigue Coefficient, $C' = 554$ psi

This data was
not provided

Referenced Documents:

- [1] RDP 1060 – 322 Load Frame Configuration for Cyclic Testing
- [2] RDP 1061 – Uniaxial Strain-Controlled Fatigue Testing

Written by:

Preview Copy

Test Summary

Table VII: Summary of constant-amplitude test results

Sample ID	Stress, σ	Reversals to Failure, $2N_f$	Remarks ^[A]	Analysis ^[B]
SEQ15	35,502	400	Load control, runout	SS E P
SEQ16	35,502	400	Load control	x
SEQ17	39,238	400	Load control	x
SEQ18	41,729	400	Load control	x
SEQ19	40,728	430	Load control	x
SEQ20	41,729	144	Load control	x x x
SEQ21	40,728	142	Load control	x x x
SEQ22	41,729	104	Load control	x x x
SEQ23	40,728	5	Load control	x x x
SEQ24	41,729	5	Load control	x x x
SEQ25	40,728	5	Load control	x x x
SEQ26	41,729	5	Load control	x x x
SEQ27	40,728	5	Load control	x x x
SEQ28	41,729	5	Load control	x x x
SEQ29	40,728	5	Load control	x x x
SEQ30	41,729	5	Load control	x x x
SEQ31	40,728	5	Load control	x x x
SEQ32	41,729	5	Load control	x x x
SEQ33	40,728	5	Load control	x x x
SEQ34	41,729	5	Load control	x x x
SEQ35	40,728	5	Load control	x x x
SEQ36	41,729	5	Load control	x x x
SEQ37	40,728	5	Load control	x x x
SEQ38	41,729	5	Load control	x x x
SEQ39	40,728	5	Load control	x x x
SEQ40	41,729	5	Load control	x x x
SEQ41	40,728	5	Load control	x x x
SEQ42	41,729	5	Load control	x x x
SEQ43	40,728	5	Load control	x x x
SEQ44	41,729	5	Load control	x x x
SEQ45	40,728	5	Load control	x x x
SEQ46	41,729	5	Load control	x x x
SEQ47	40,728	5	Load control	x x x
SEQ48	41,729	5	Load control	x x x
SEQ49	40,728	5	Load control	x x x
SEQ50	41,729	5	Load control	x x x
SEQ51	40,728	5	Load control	x x x
SEQ52	41,729	5	Load control	x x x
SEQ53	40,728	5	Load control	x x x
SEQ54	41,729	5	Load control	x x x
SEQ55	40,728	5	Load control	x x x
SEQ56	41,729	5	Load control	x x x
SEQ57	40,728	5	Load control	x x x
SEQ58	41,729	5	Load control	x x x
SEQ59	40,728	5	Load control	x x x
SEQ60	41,729	5	Load control	x x x
SEQ61	40,728	5	Load control	x x x
SEQ62	41,729	5	Load control	x x x
SEQ63	40,728	5	Load control	x x x
SEQ64	41,729	5	Load control	x x x
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SEQ66	41,729	5	Load control	x x x
SEQ67	40,728	5	Load control	x x x
SEQ68	41,729	5	Load control	x x x
SEQ69	40,728	5	Load control	x x x
SEQ70	41,729	5	Load control	x x x
SEQ71	40,728	5	Load control	x x x
SEQ72	41,729	5	Load control	x x x
SEQ73	40,728	5	Load control	x x x
SEQ74	41,729	5	Load control	x x x
SEQ75	40,728	5	Load control	x x x
SEQ76	41,729	5	Load control	x x x
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SEQ78	41,729	5	Load control	x x x
SEQ79	40,728	5	Load control	x x x
SEQ80	41,729	5	Load control	x x x
SEQ81	40,728	5	Load control	x x x
SEQ82	41,729	5	Load control	x x x
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SEQ91	40,728	5	Load control	x x x
SEQ92	41,729	5	Load control	x x x
SEQ93	40,728	5	Load control	x x x
SEQ94	41,729	5	Load control	x x x
SEQ95	40,728	5	Load control	x x x
SEQ96	41,729	5	Load control	x x x
SEQ97	40,728	5	Load control	x x x
SEQ98	41,729	5	Load control	x x x
SEQ99	40,728	5	Load control	x x x
SEQ100	41,729	5	Load control	x x x

See Ref 2. for procedures regarding strain-controlled

[A] Complete Remarks can be found in the associated

[B] The Analysis columns identify which specimen

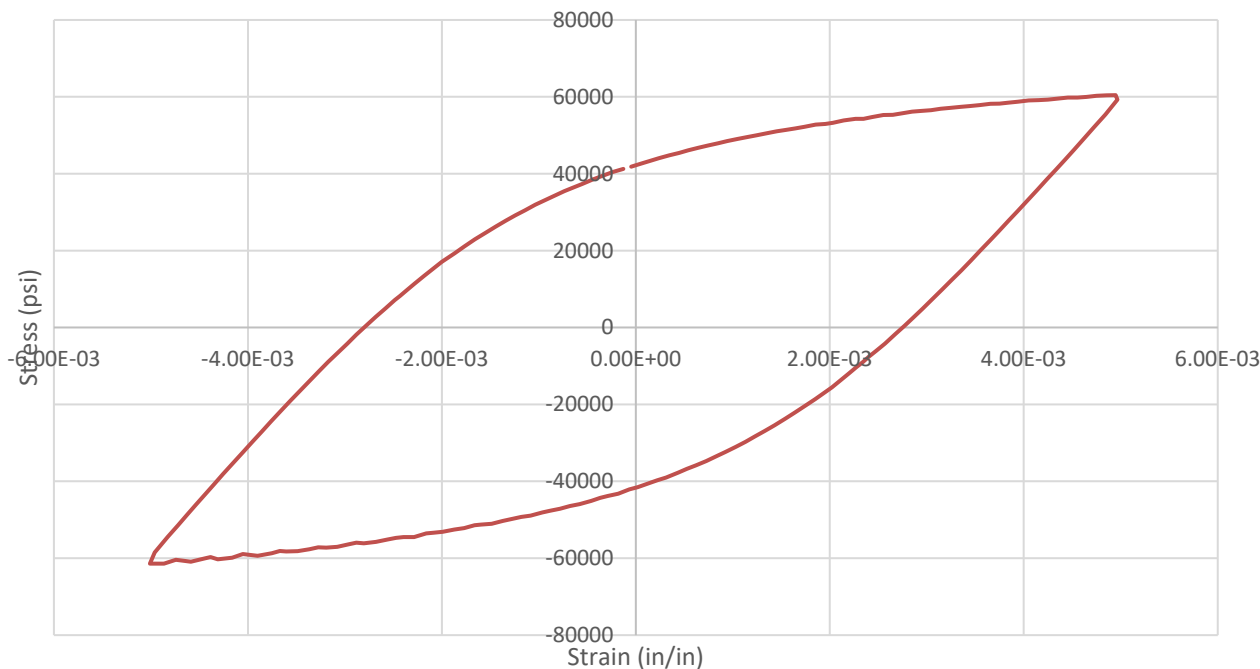
SS, Cyclic Stress-Strain Curve; E, Elastic Strain

be found in the associated companion test spread

This loop and point
was provided

ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman MAR-5589a – Gr 50...

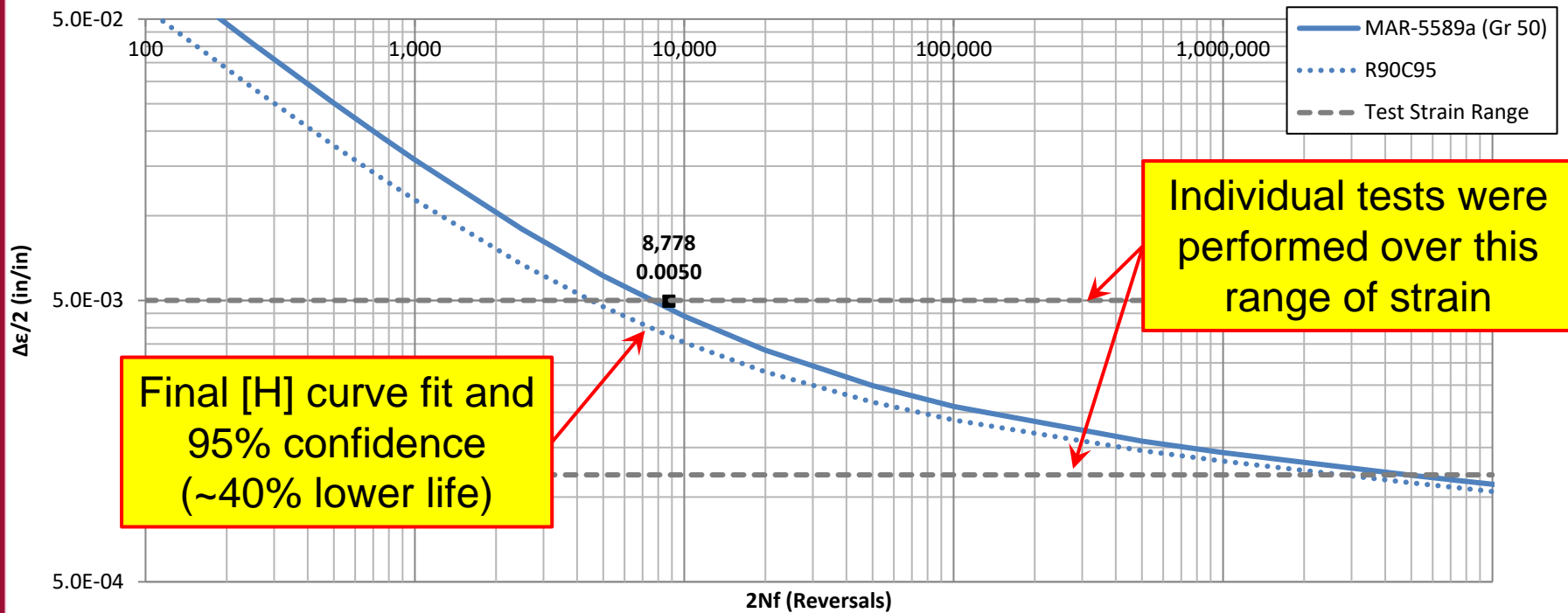
SEQ08-13; Cycle 2000; Nf = 4389 (GR 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-5589a – Gr 50...

MAR-5589a (Gr 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. H MAR-5589a – Gr 50...

Re: e-N Prediction...

PH Peter Huffman <huffman.peter.j@gmail.com>
To ✓ Patterson, James

WARNING: EXTERNAL EMAIL!

Hi Jim,

For the SEQ08:

E	28.17MSI
n'	0.115
K'	107.7KSI
S _F '	120.2KSI
b	-0.0855
e _F '	2.594
c	-0.7435
UTS	76.31KSI

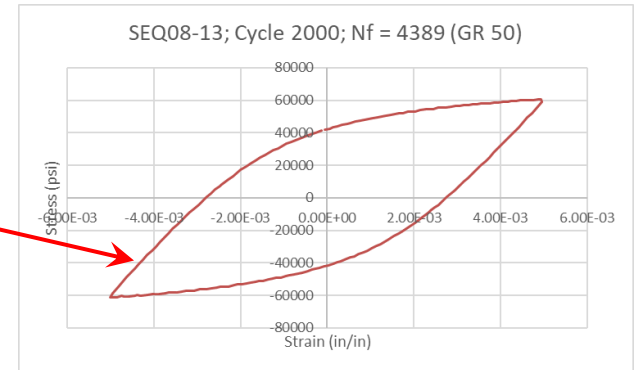
and for the SEQ1:

E	31.16MSI
n'	0.1105
K'	155.4KSI
S _F '	160.1KSI
b	-0.0830
e _F '	1.307
c	-0.7510
UTS	111.64KSI

Sorry for the late reply, today has been nuts!

Prediction from single
hysteresis loop

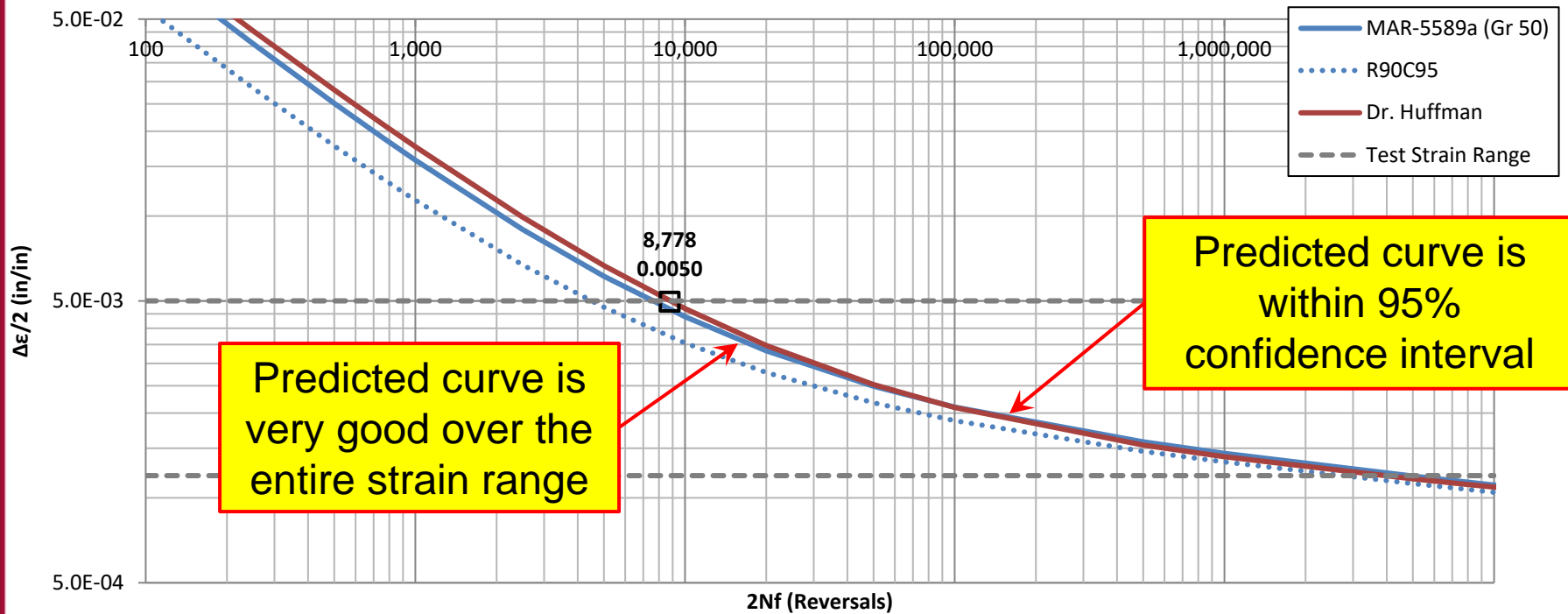
UTS Prediction



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-5589a – Gr 50...

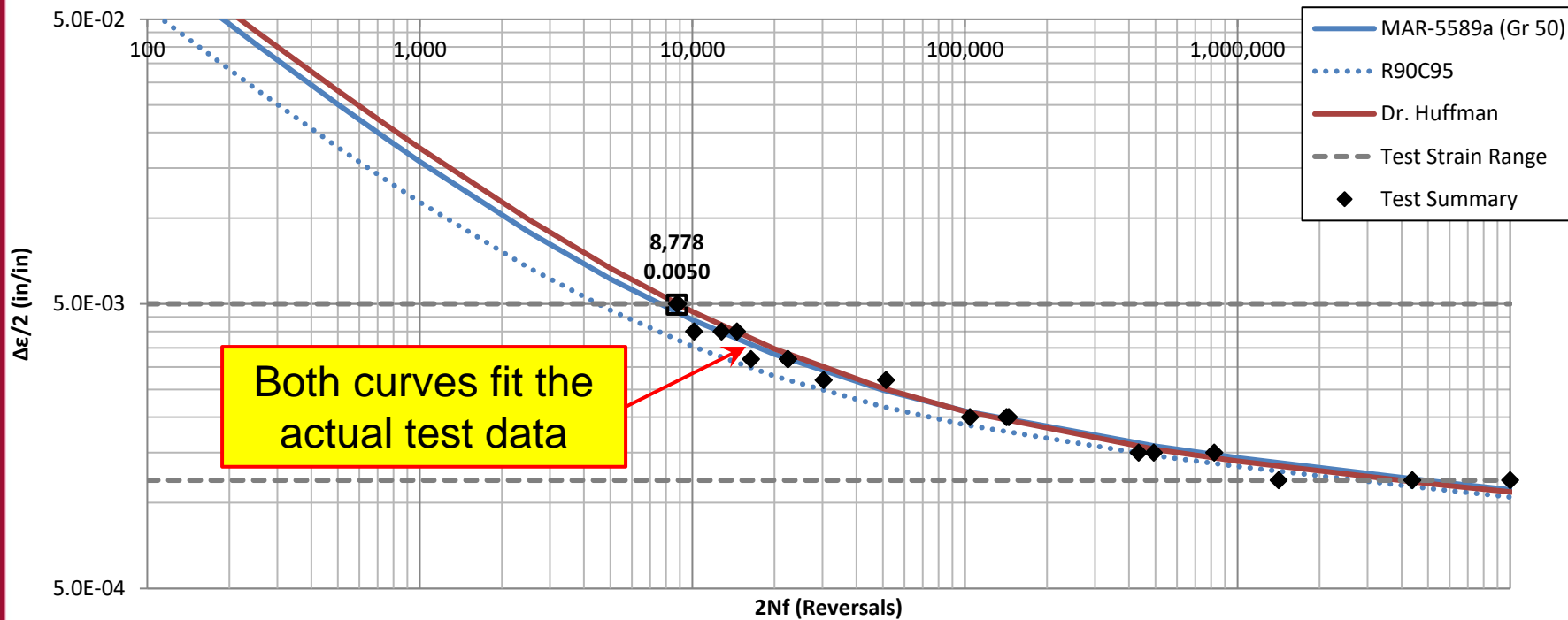
MAR-5589a (Gr 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

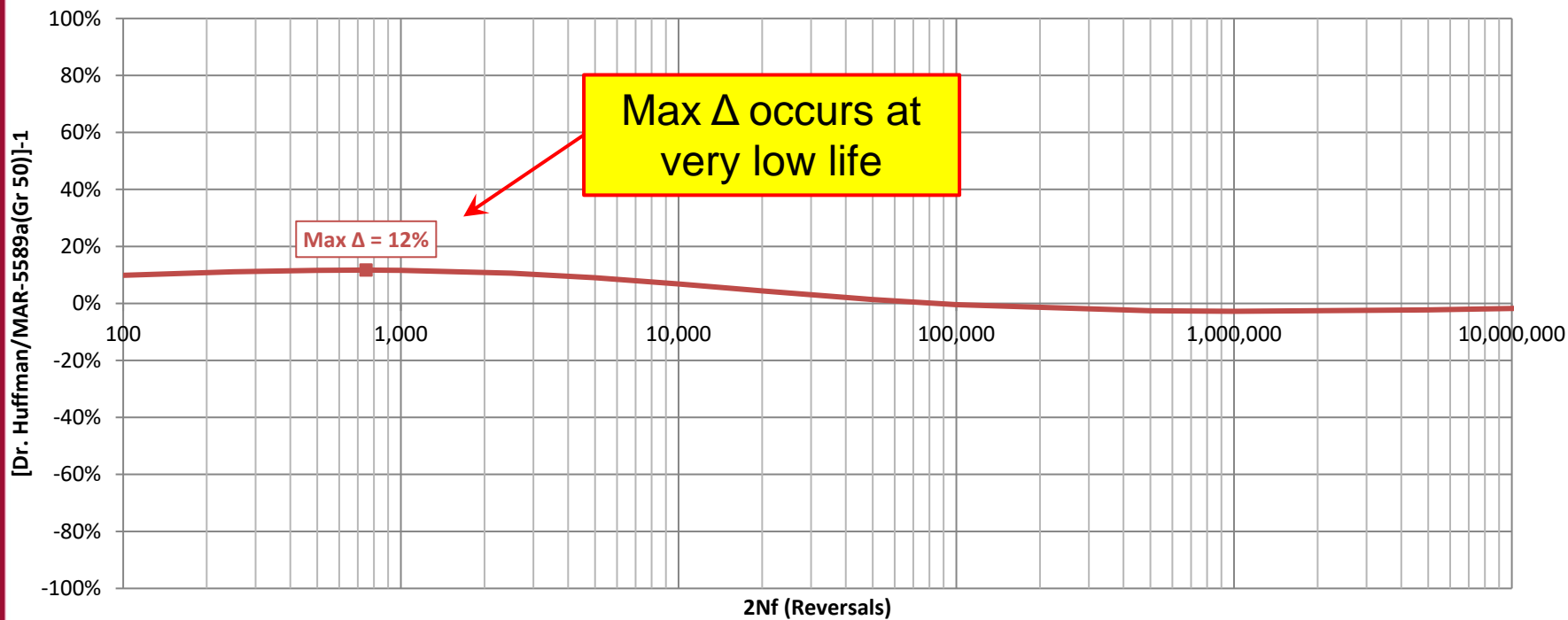
MAR-5589a – Gr 50...

MAR-5589a (Gr 50)



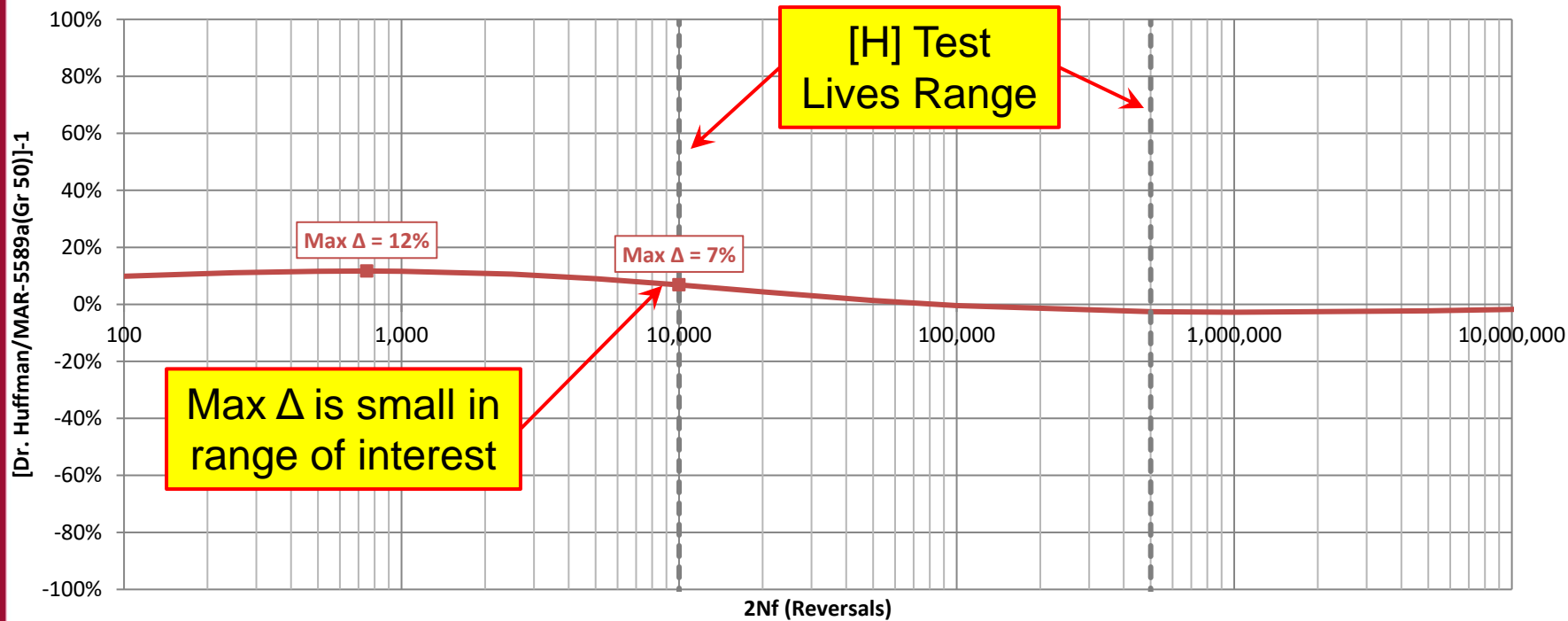
ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman MAR-5589a – Gr 50...

MAR-5589a (Gr 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman MAR-5589a – Gr 50...

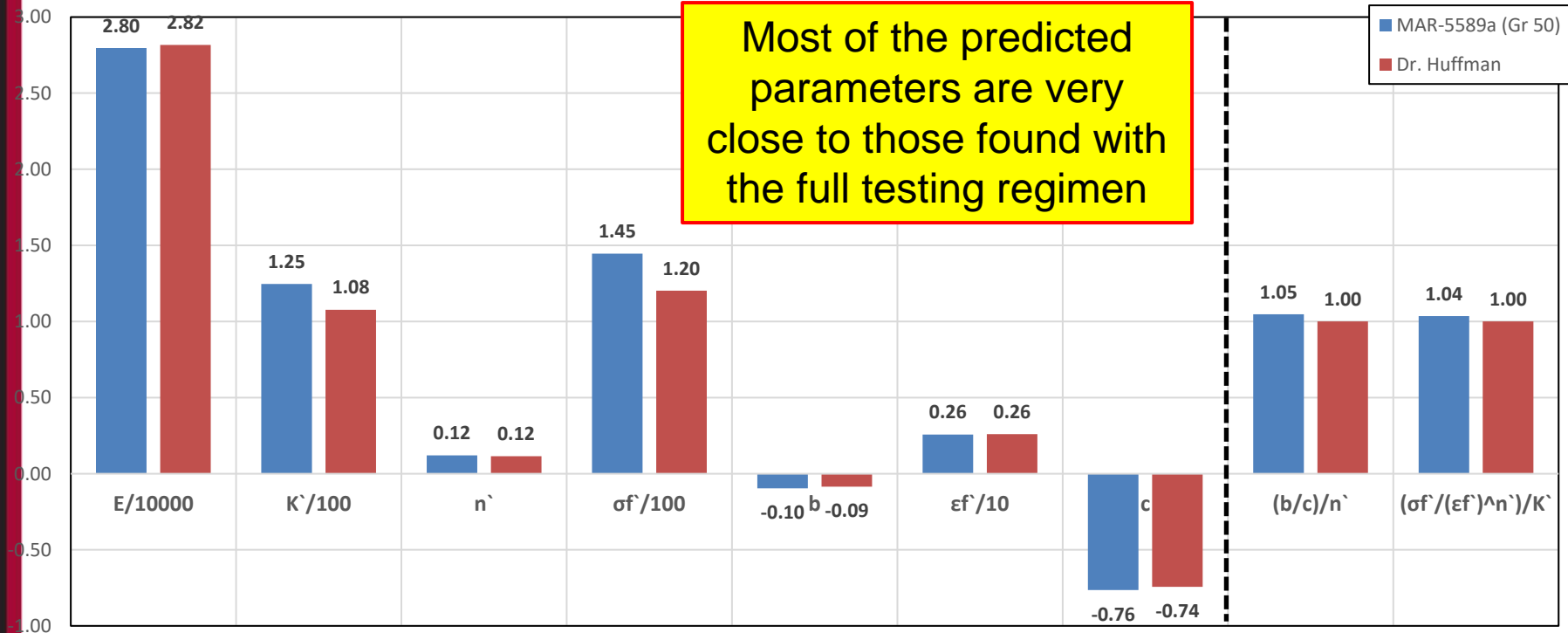
MAR-5589a (Gr 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-5589a – Gr 50...

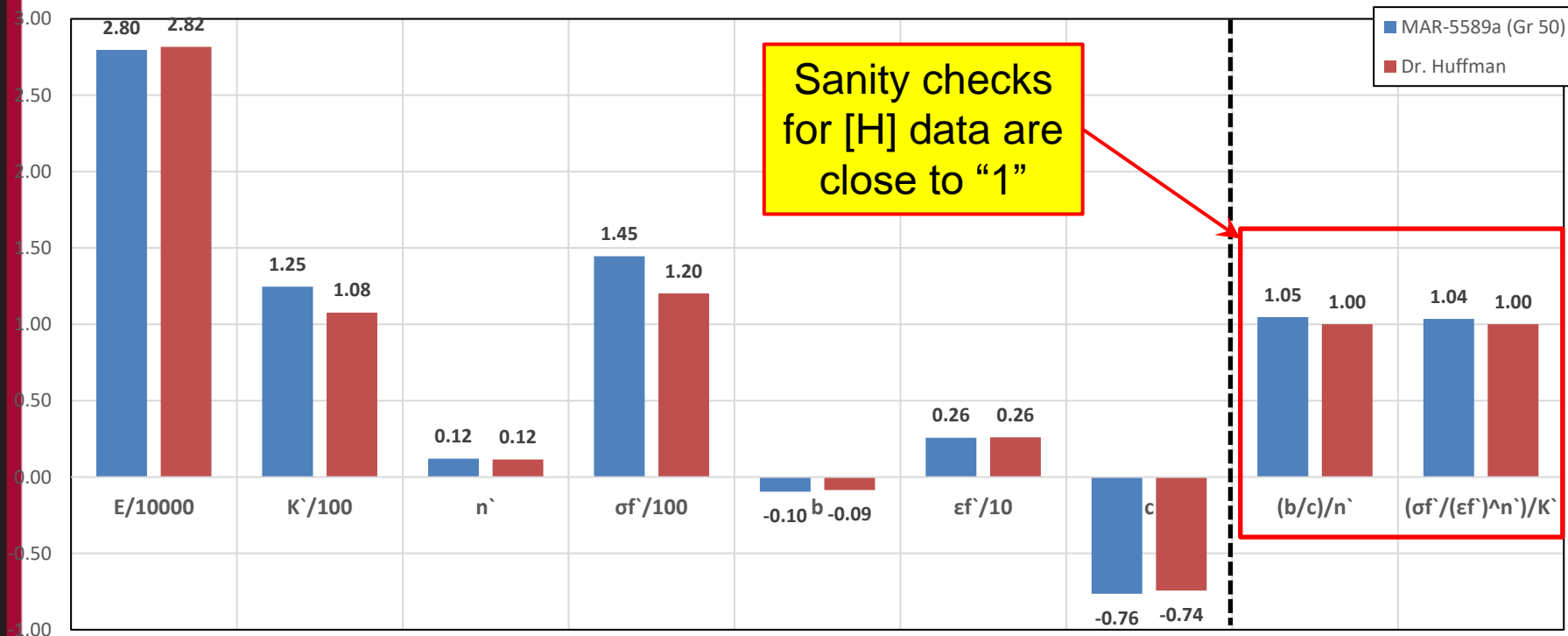
Parameter Comparison



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-5589a – Gr 50...

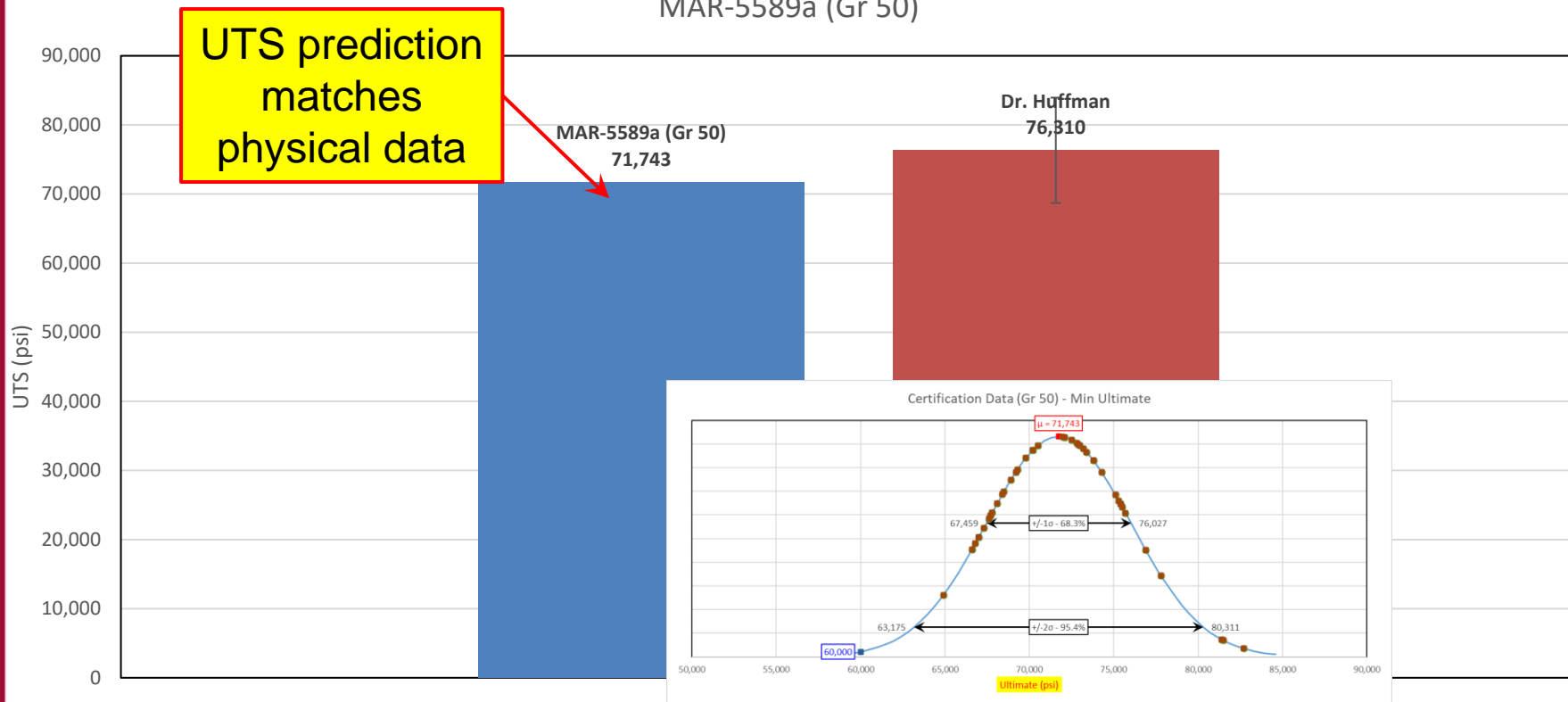
Parameter Comparison



ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-4108a – Gr 80...

MAR-5589a (Gr 50)



ϵ -N Curve Prediction (Hyst. Loop): Dr. H

MAR-5589a – Gr 50...

■ Conclusions...

- To test the method, MAR data was provided to Dr. Huffman...
 - ◆ MAR-5589a – Gr 50 Plate
 - ◆ Single hysteresis loop and corresponding $2N_f$.
- The predicted curve...
 - ◆ Matched well (~12%) over the entire strain range.
 - ◆ Matched very well (~7%) within the normal range of [H] testing.
 - ◆ Was within the 95% Confidence Interval over the entire strain range.
- The Predicted Parameters...
 - ◆ Were in general very close to the test predictions.
 - ◆ The [H] data sanity checks were close to one.

***Strain-Life Curve Prediction
from Hysteresis Loops
MAR-4127a r1 – D65-45-12***

ε-N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-4127aR1 – D65-45-12....

HENDRICKSON

TRAILER COMMERCIAL
VEHICLE SYSTEMS

To J. PATTERSON – PRINCIPAL VEHICLE SYSTEM ENGINEER

From A. YOURIL – METALLURGICAL ENGINEER File. MAR-4127A R1

Subject CYCLIC PROPERTIES OF 65-45-12 DUCTILE IRON HUBS – KIC & GUNITE Date 1 November 2016

This report to cover one subject only

TEST PROGRAM SUMMARY

To characterize the cyclic properties of ASTM A536 Grade 65-45-12 ductile iron hub castings, a fatigue test program was performed according to the recommendations of ASTM E606. Samples were obtained from production hub castings from two manufacturers, KIC (four hubs, C-33387) and Gunite (two hubs, C-28366). Clear differences were observed manufacturer to manufacturer in terms of microstructural appearance and monotonic properties, but the fatigue results could not be differentiated statistically. As such, the fatigue parameters below were produced by pooling the data sets together into a single stress-strain curve. For information, the data are presented separately in the Appendix.

Cyclic Stress-Strain	$\frac{\Delta \epsilon}{2} = \frac{\Delta \sigma}{2E^*} + \left(\frac{\Delta \sigma}{2K'}\right)^{1/n'}$	Cyclic Strain	Exponent, $n' = 12$	689 psi
Elastic Strain-Life	$\frac{\Delta \epsilon_e}{2} = \frac{\sigma'_f}{E^*} (2N_f)^b$	Fatigue Length Coefficient	$\sigma'_f = 12$	psi
Plastic Strain-Life	$\frac{\Delta \epsilon_p}{2} = \epsilon'_f (2N_f)^c$	Fatigue Ductility Coefficient	$\epsilon'_f = 0.931$	in/in

This data was not provided

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Table V: Summary of constant-amplitude test results

Sample ID	Strain, $\Delta \epsilon/2$ (in/in)	Stress, $\Delta \sigma/2$ (ksi)	Reversals to Failure, $2N_f$	Remarks ^[A]	Analysis ^[B] SS E P
K1-5	0.004	29,611	10,000,000		
K1-7		33,577	10,000,000		
K2-16		28,894	2,661,730		
K3-2		24,894	2,478,912		
G1-2		724,852		Slight cold	
G3-27	0.00137	592,726			
G3-23	0.00137	459,756			
K4-34	0.00156	49,980		Slight tensile	
G1-2	0.00137				
G3-27	0.00137	42,730			
G3-23	0.00149	37,333			
K4-34	0.00178	43,255			
G1-2	0.00181	44,162			
G3-27	0.00181	44,498			
G3-23	0.00181	44,498	51	zeroed	
K4-34	0.00203	42,720			
G1-2	0.00203				
G3-27	0.00203	55,368	10,046	Slight mean compressive stress	
G3-23	0.00221	50,126	10,040		
K4-29	0.00300	59,277	9,410		
K3-32	0.00263	56,946	9,090		
K1-9	0.00263	55,680	8,006		
K2-14	0.00300				
K1-4	0.00250				
K2-18	0.00263				
G1-6	0.00269				
G1-10	0.00328				
G3-30	0.00300	55,200	1,340		
G1-15	0.00300	54,922	1,240		
K2-3	0.00310	54,176	1,052	Diam and E corrected (Mic)	
K2-1	0.00470	61,903	740	Diam and E corrected (Mic)	

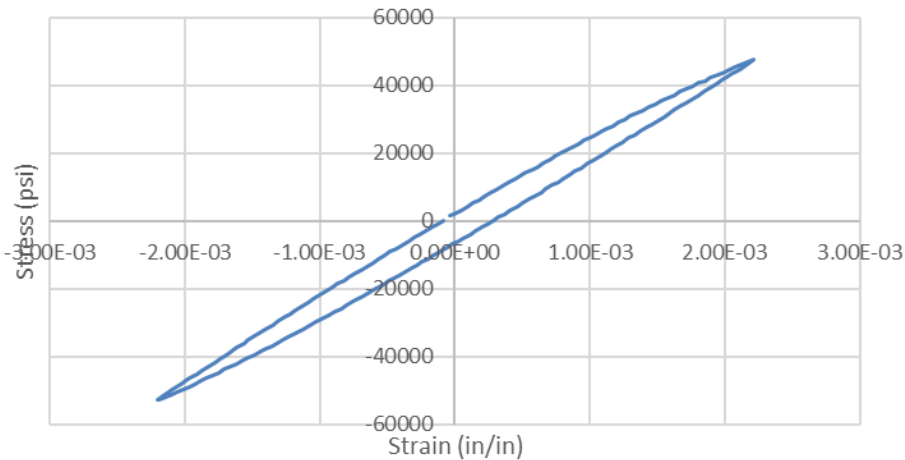
These loops and points were provided

ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-4127aR1 – D65-45-12...

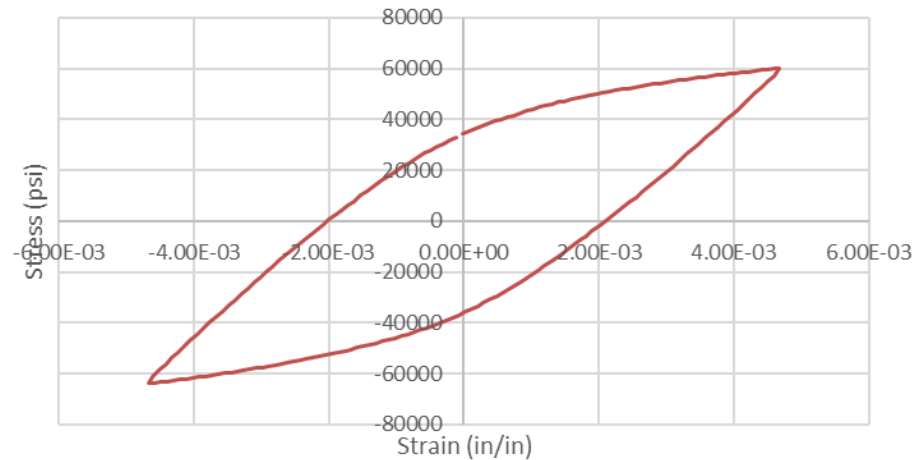
G1-8	0.00221	50,126	10,040
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G1-8; Cycle 3000; Nf = 5020



K2-1	0.00470	61,903	740
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K2-1; Cycle 300; Nf = 370

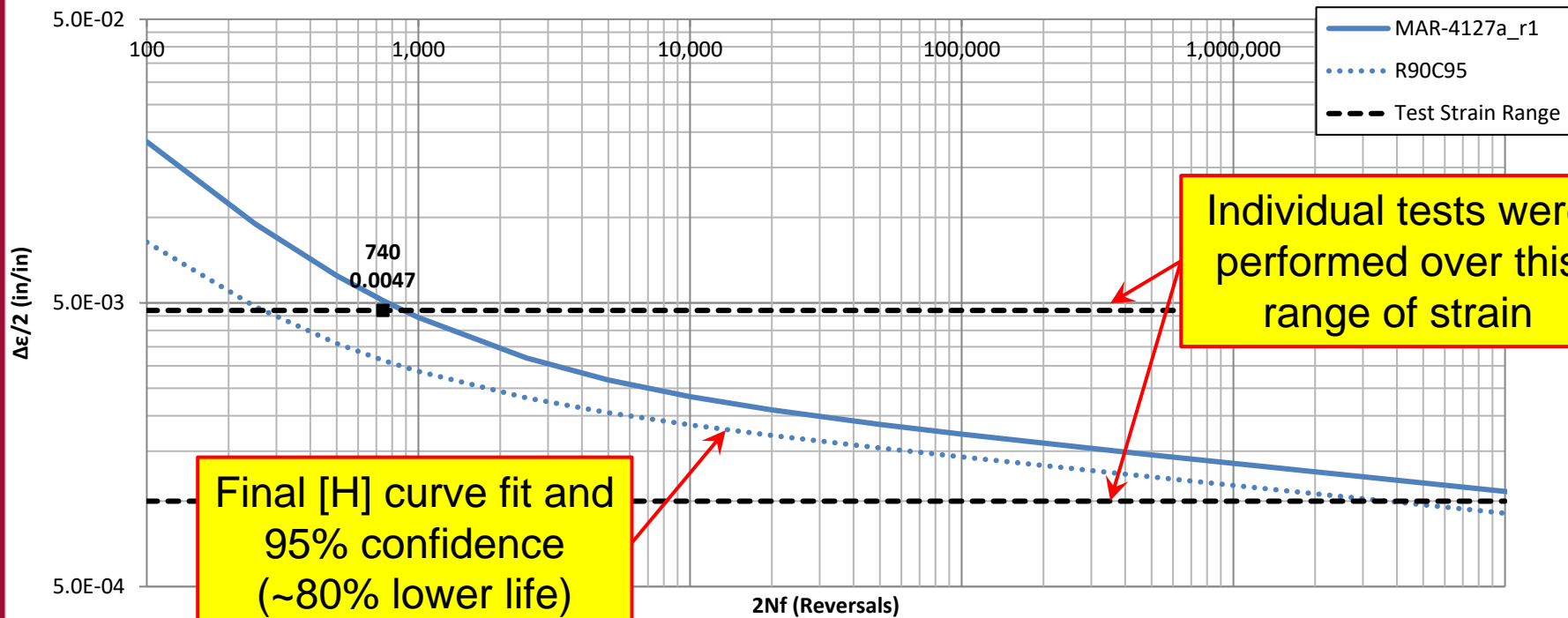


Two loops with different strain levels.
What level of hysteresis is needed for
a good prediction?

ϵ -N Curve Prediction (Hyst. Loop): Dr. Huffman

MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

Re: e-N Prediction...



Peter Huffman <huffman.peter.j@gmail.com>

To ✓ Patterson, James

Reply

Reply All

Forward

...

Thu 9/17/2020 2:44 PM

You replied to this message on 9/17/2020 3:39 PM.

WARNING:EXTERNAL EMAIL!

So far I've only looked at the 300 cycle loop. I have not opened the other two files. If my hysteresis calculation is working correctly,

$E = 25.69$ Mpsi

which leads me to believe this is some kind of stainless steel. If that's not the right modulus, probably everything else will be off too.

$n' = 0.1356$

$K' = 130$ ksi

$Sf' = 108.8$ ksi

$b = -0.0964$

$ef' = 0.266$

$c = -0.7108$

Prediction from loop
with larger hysteresis

the b/c etc. can be found here, but it's just mentioned:

https://www.efatigue.com/training/Chapter_5.pdf

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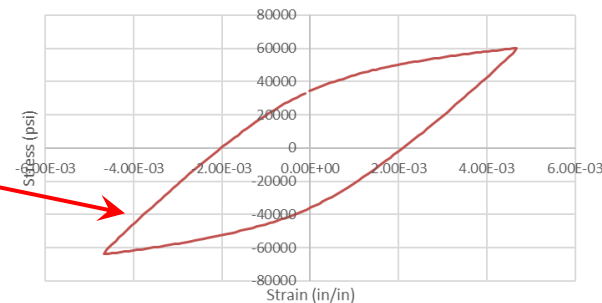
The K' part of the relationship is mentioned in SAE "Proposed Technical Report on Fatigue Properties for the SAE Handbook" from 1974 But at a glance I'm not seeing the b/c bit

<https://www.sae.org/publications/technical-papers/content/740279/>

and if you're feeling really peppy, it's not too hard to derive.

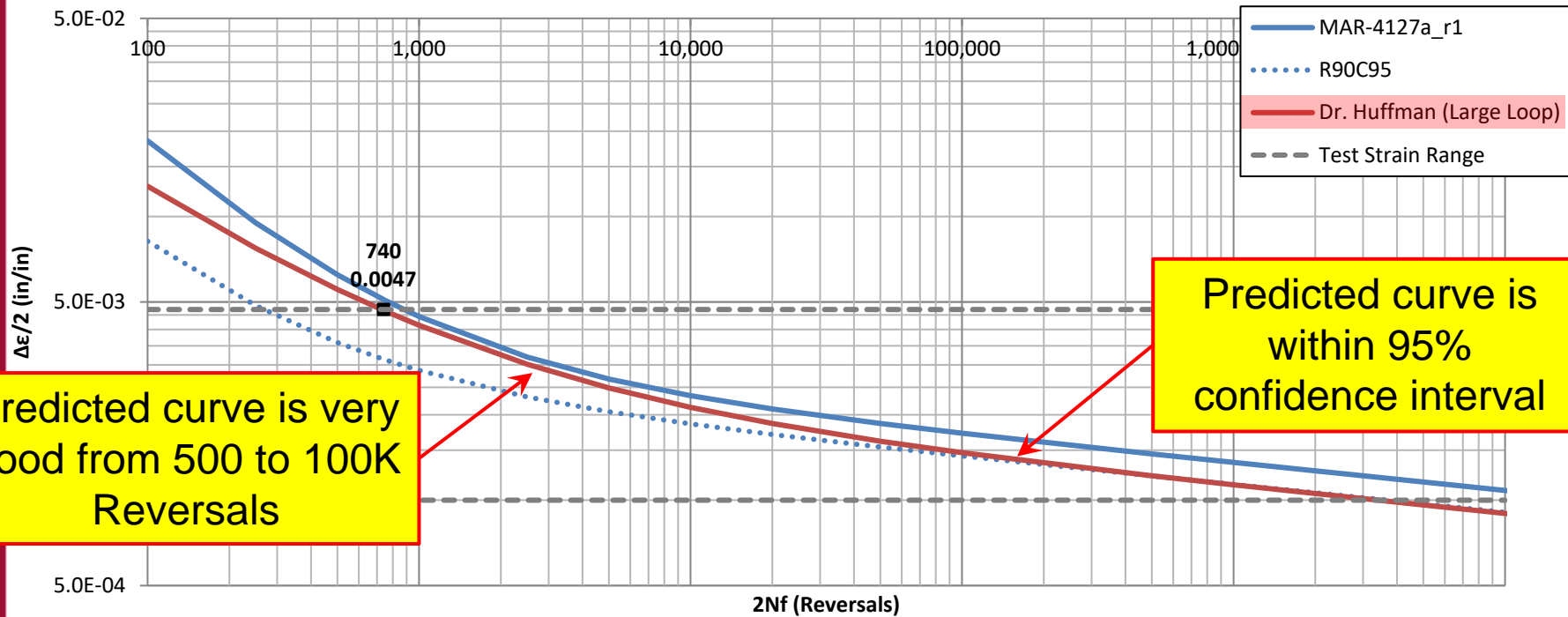
I'll look forward to hearing back. It's really hard not to open up that material reveal file... :-)

K2-1; Cycle 300; $N_f = 370$



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

MAR-4127a_r1

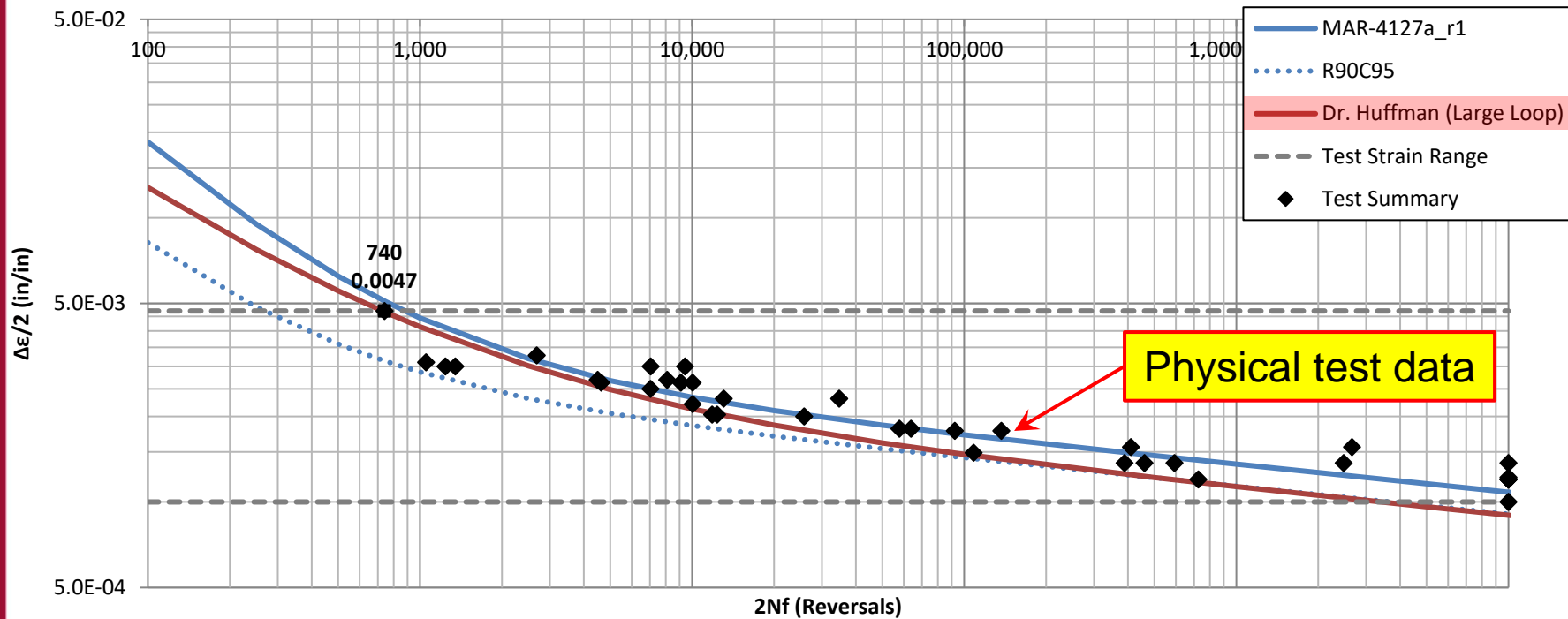


Predicted curve is very good from 500 to 100K Reversals

Predicted curve is within 95% confidence interval

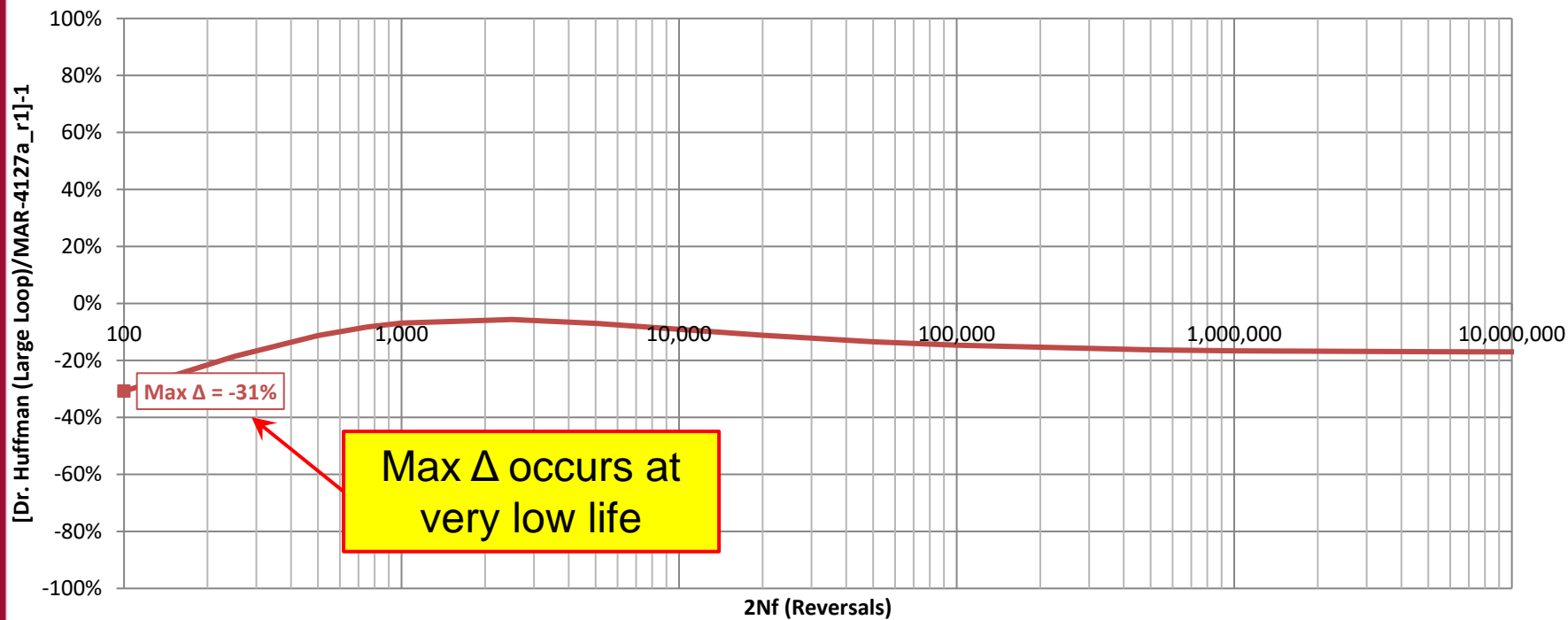
ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



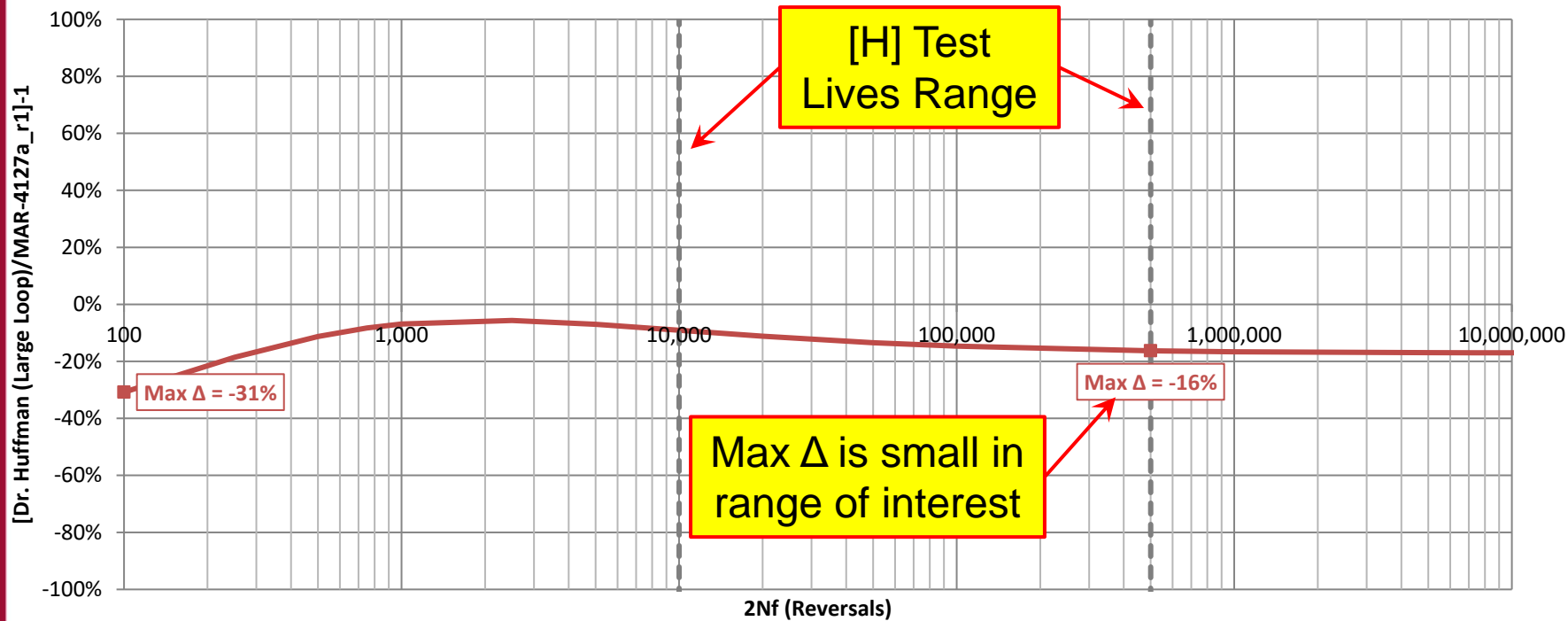
ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

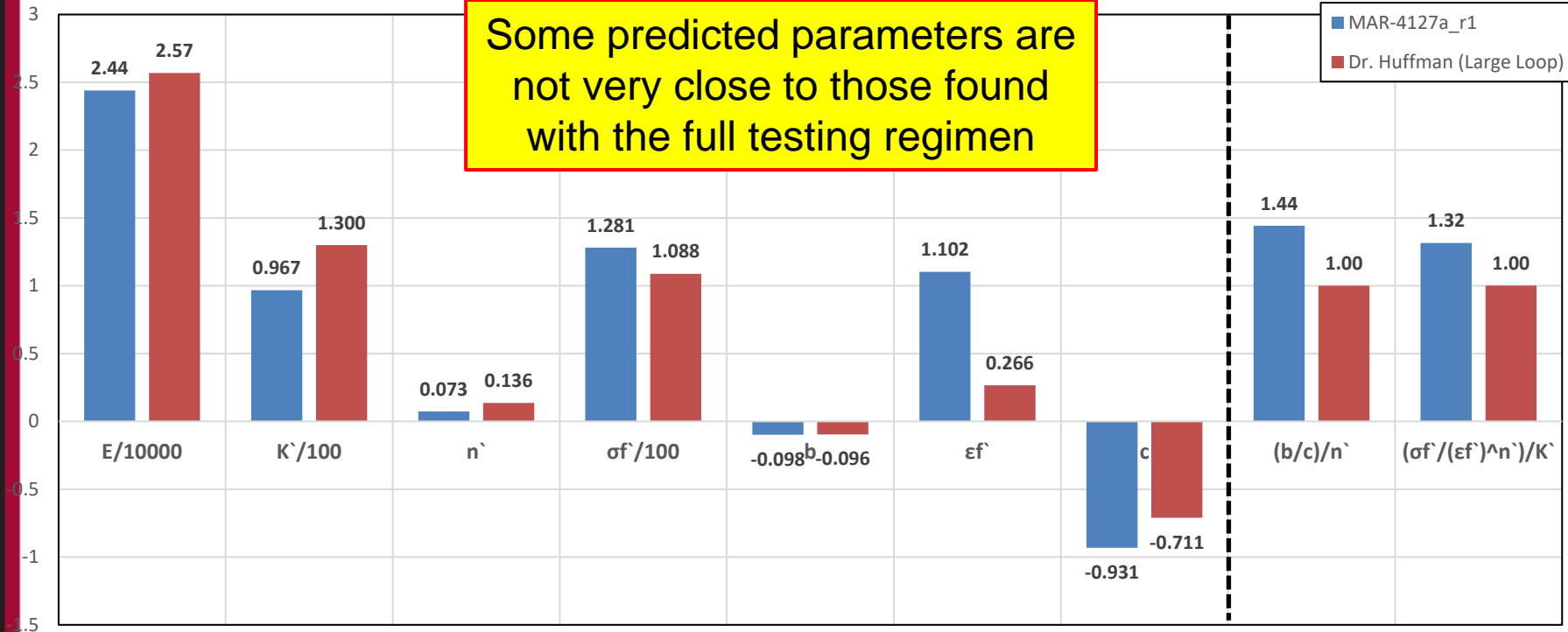
MAR-4127a_r1



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

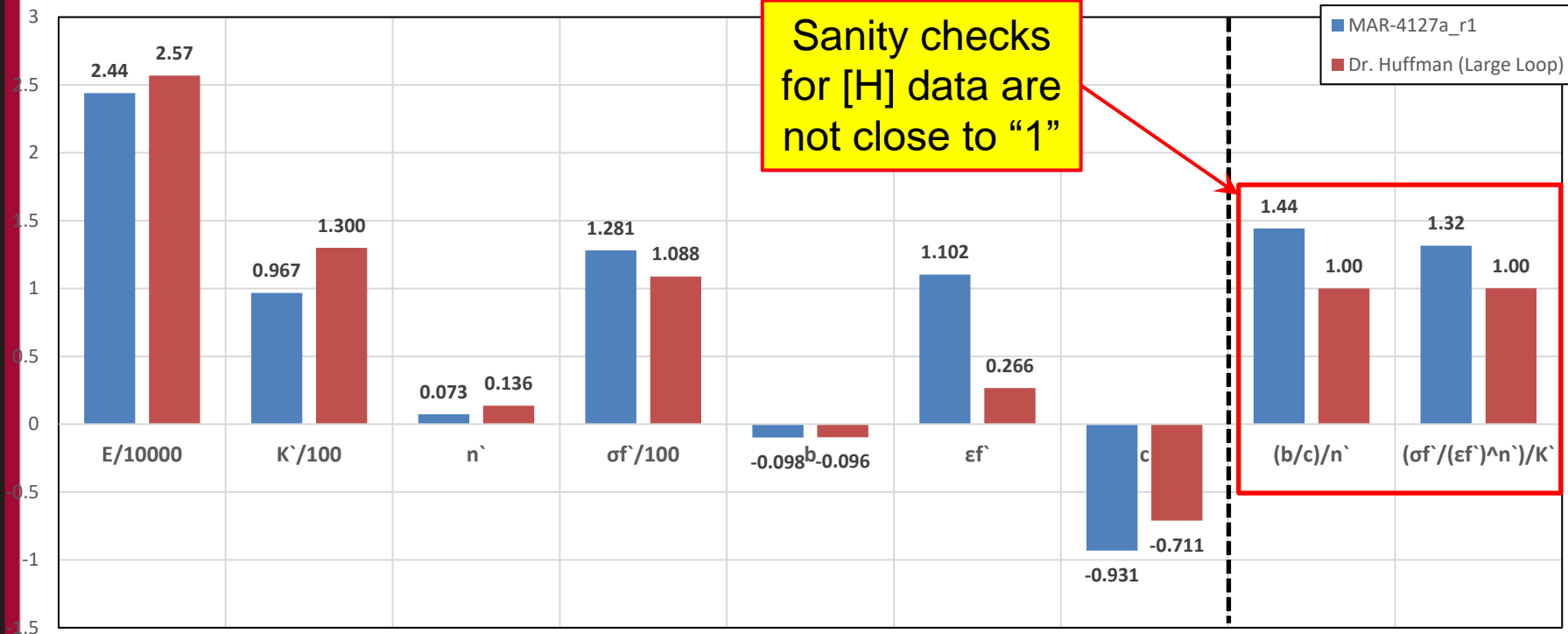
Parameter Comparison

Some predicted parameters are not very close to those found with the full testing regimen



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

Parameter Comparison



ϵ -N Curve Prediction (Large Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

■ Conclusions...

- To test the method, MAR data was provided to Dr. Huffman...
 - ◆ MAR-4127a r1 – D65-45-12
 - ◆ Large hysteresis loop and corresponding $2N_f$.
- The predicted curve...
 - ◆ Matched somewhat (~31%) over the entire strain range.
 - ◆ Matched well (~16%) within the normal range of [H] testing.
 - ◆ Was within the 95% Confidence Interval over the entire strain range.
- The Predicted Parameters...
 - ◆ Showed some variation.
 - ◆ The [H] data sanity checks were not close to one.

ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

Re: e-N Prediction...



Peter Huffman <huffman.peter.j@gmail.com>
To: ✓ Patterson, James

Reply

Reply All

Forward

...

Thu 9/17/2020 4:43 PM

WARNING: EXTERNAL EMAIL!

using only the small loop, I get $E = 26.3$ Mpsi

$n' = 0.3173$

$K' = 63.1$ ksi

$Sf' = 221$ ksi

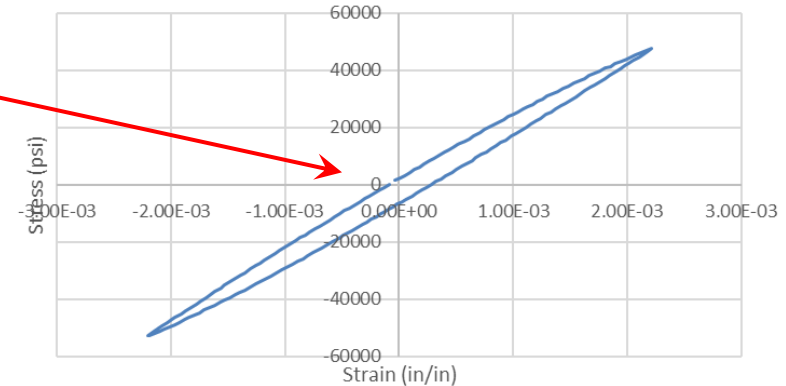
$b = -0.1626$

$ef = 0.037$

$c = -0.5123$

Prediction from loop
with smaller hysteresis

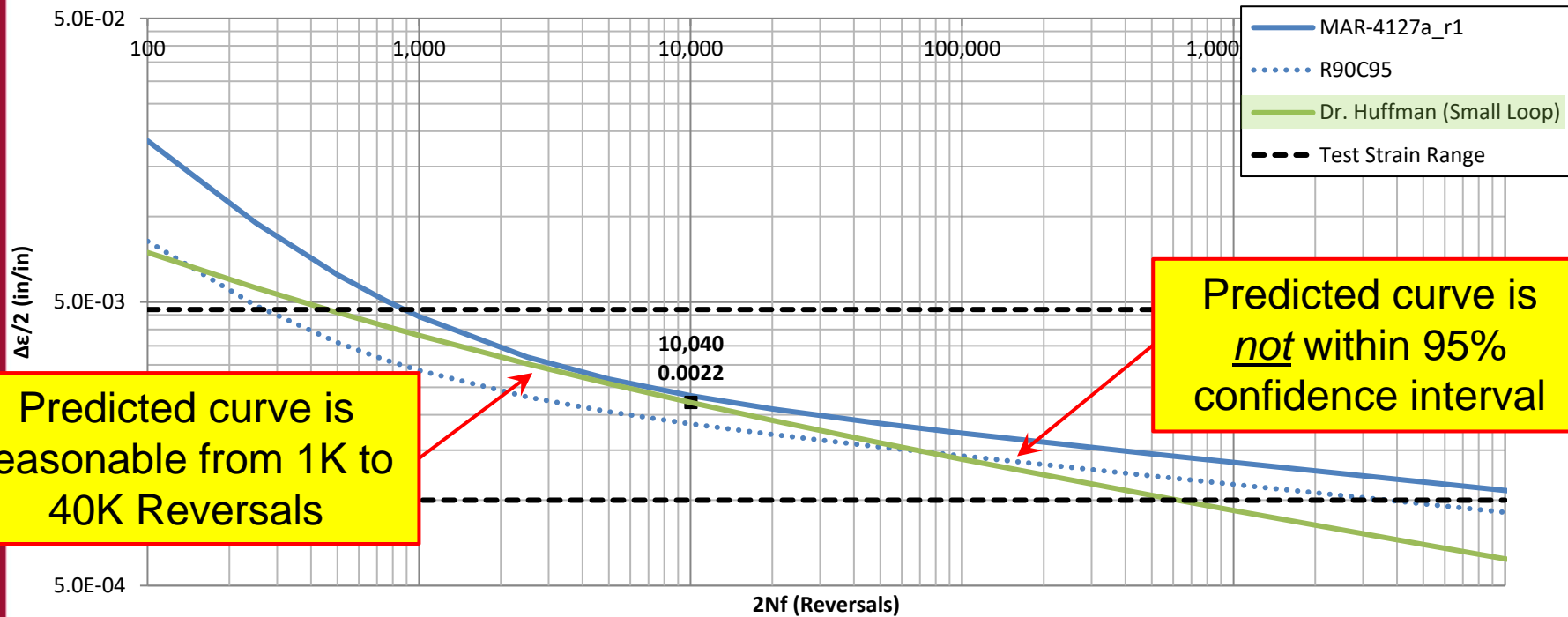
G1-8; Cycle 3000; $N_f = 5020$



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



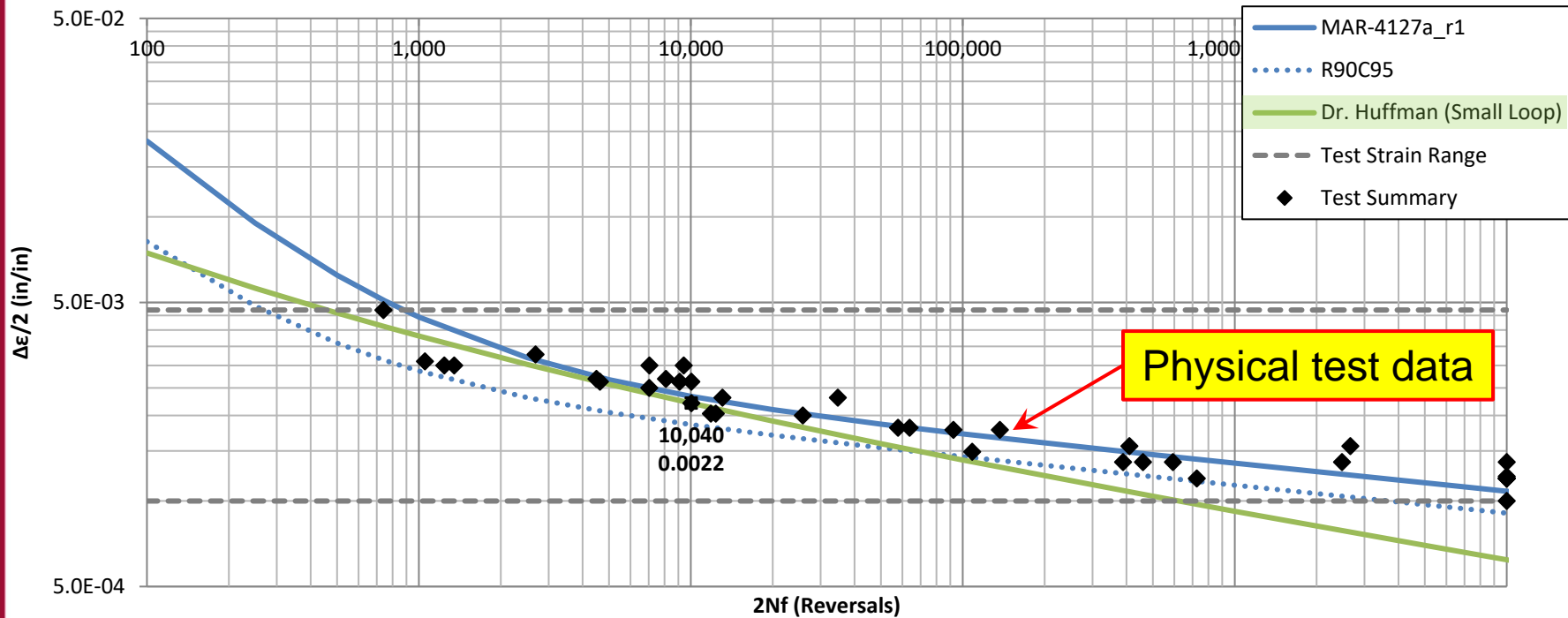
Predicted curve is reasonable from 1K to 40K Reversals

Predicted curve is not within 95% confidence interval

ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

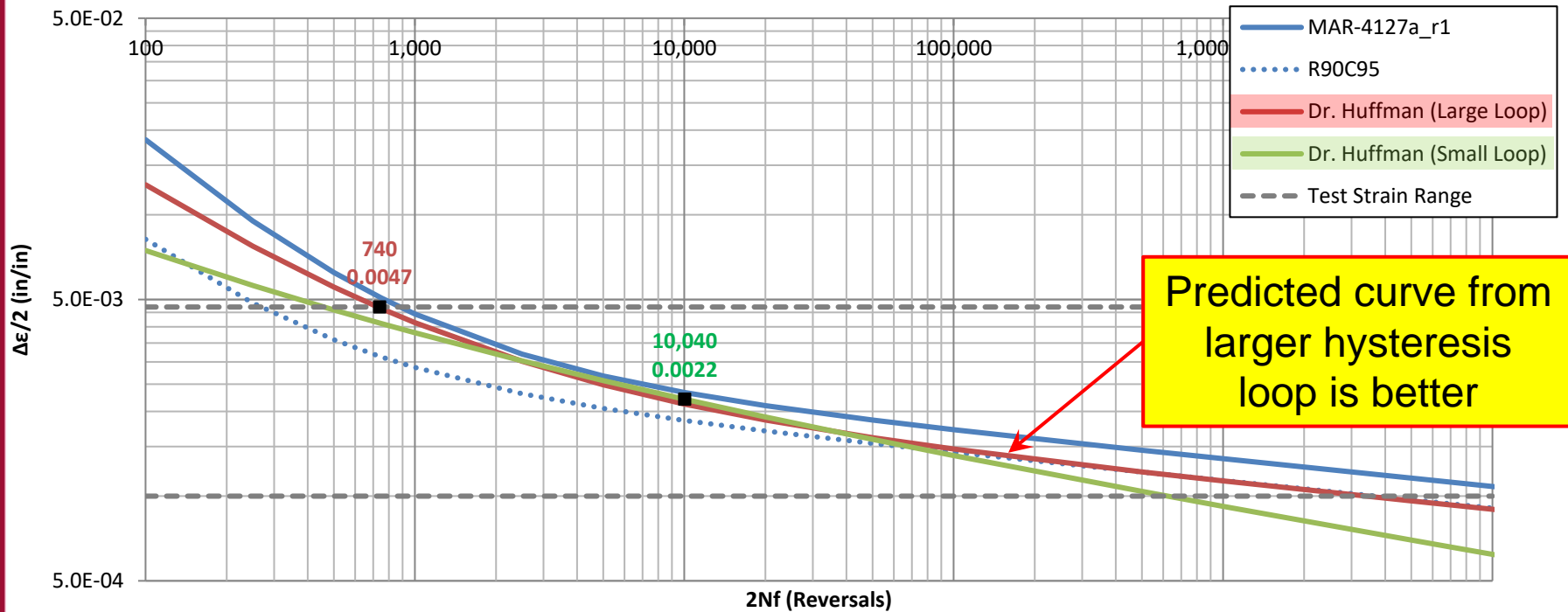
MAR-4127a_r1



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

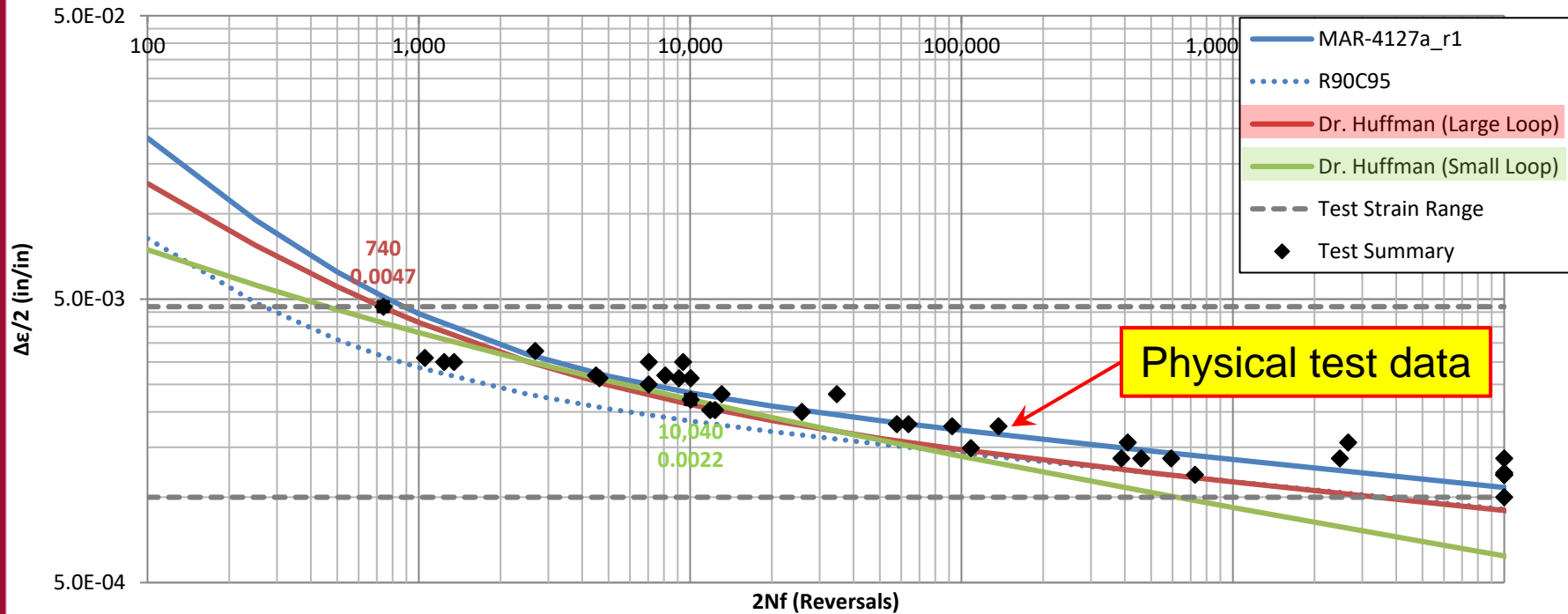
MAR-4127a_r1



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

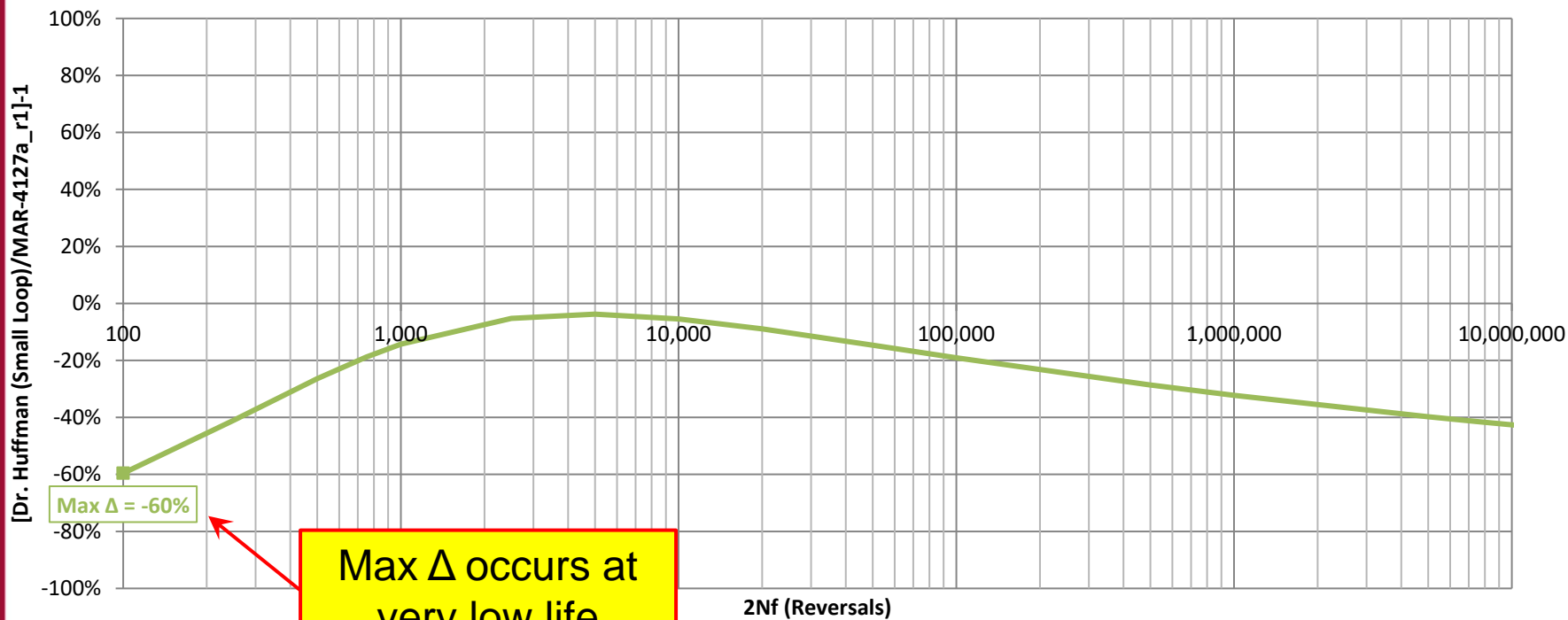
MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



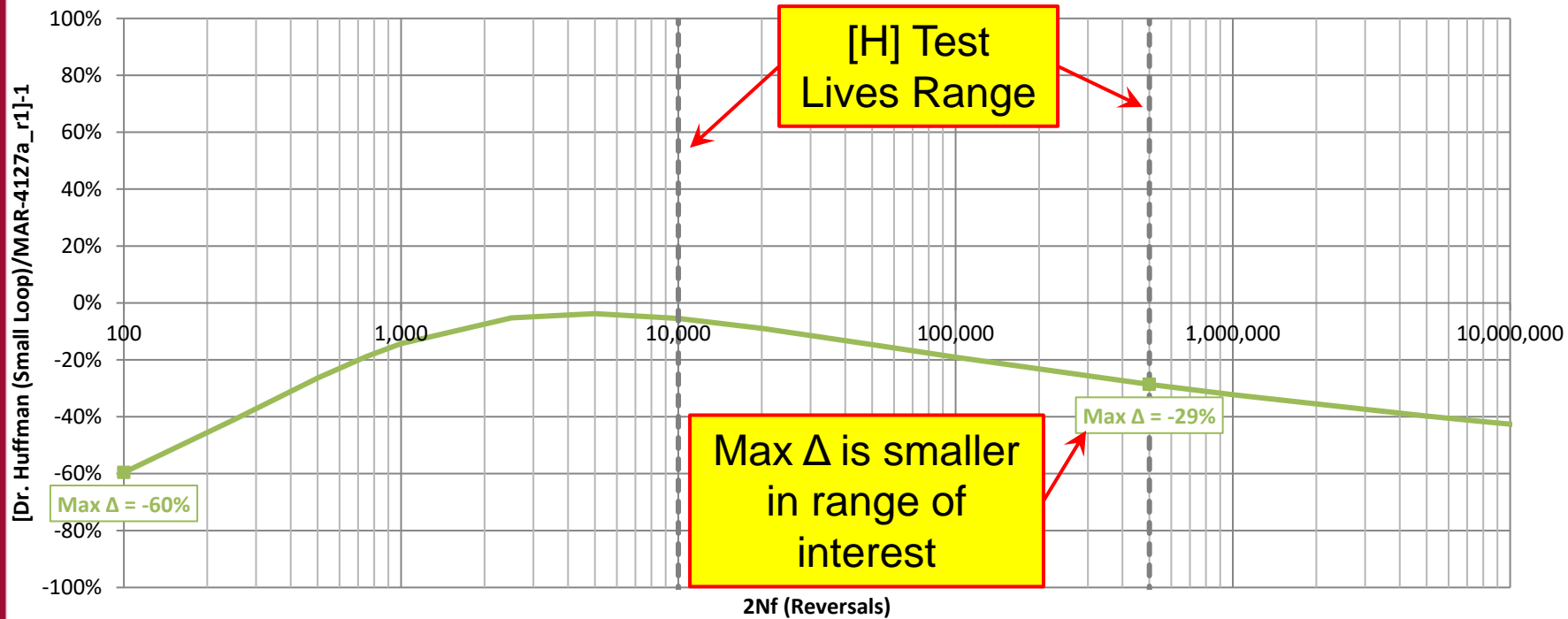
ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

MAR-4127a_r1



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

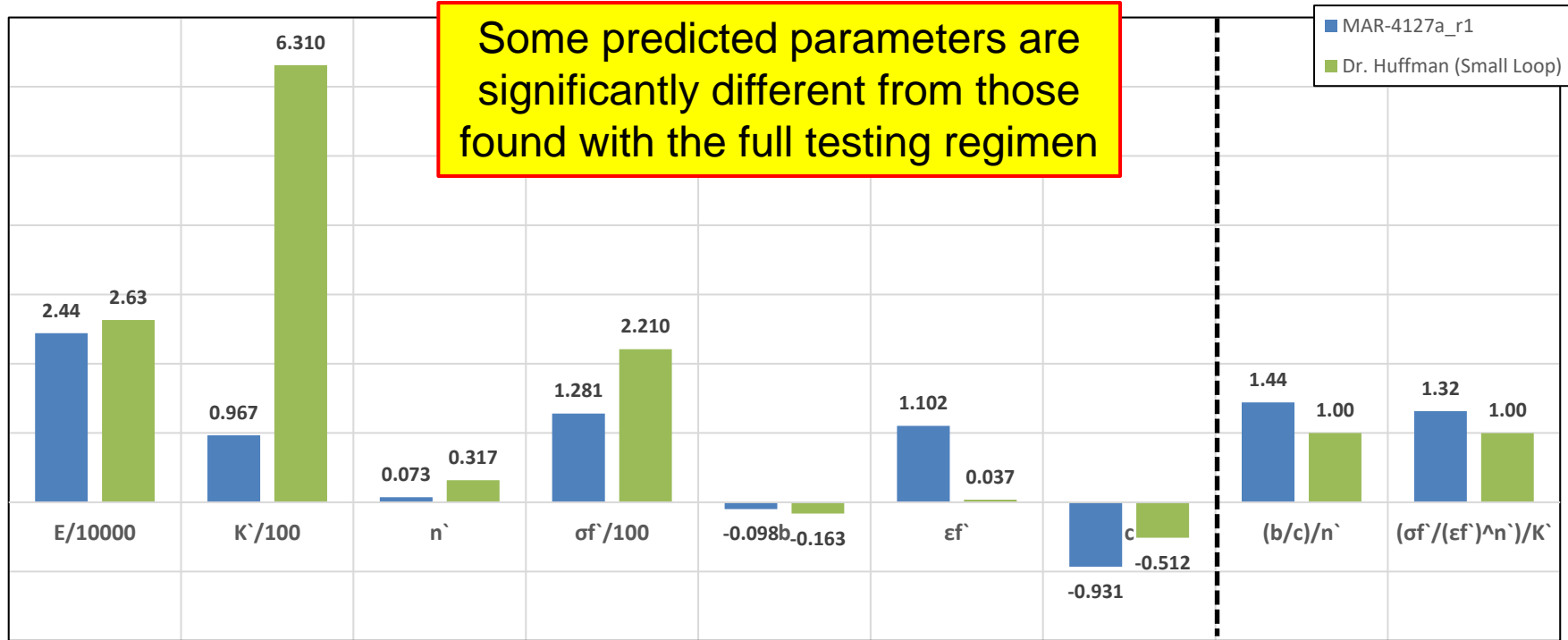
MAR-4127a_r1



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

MAR-4127aR1 – D65-45-12...

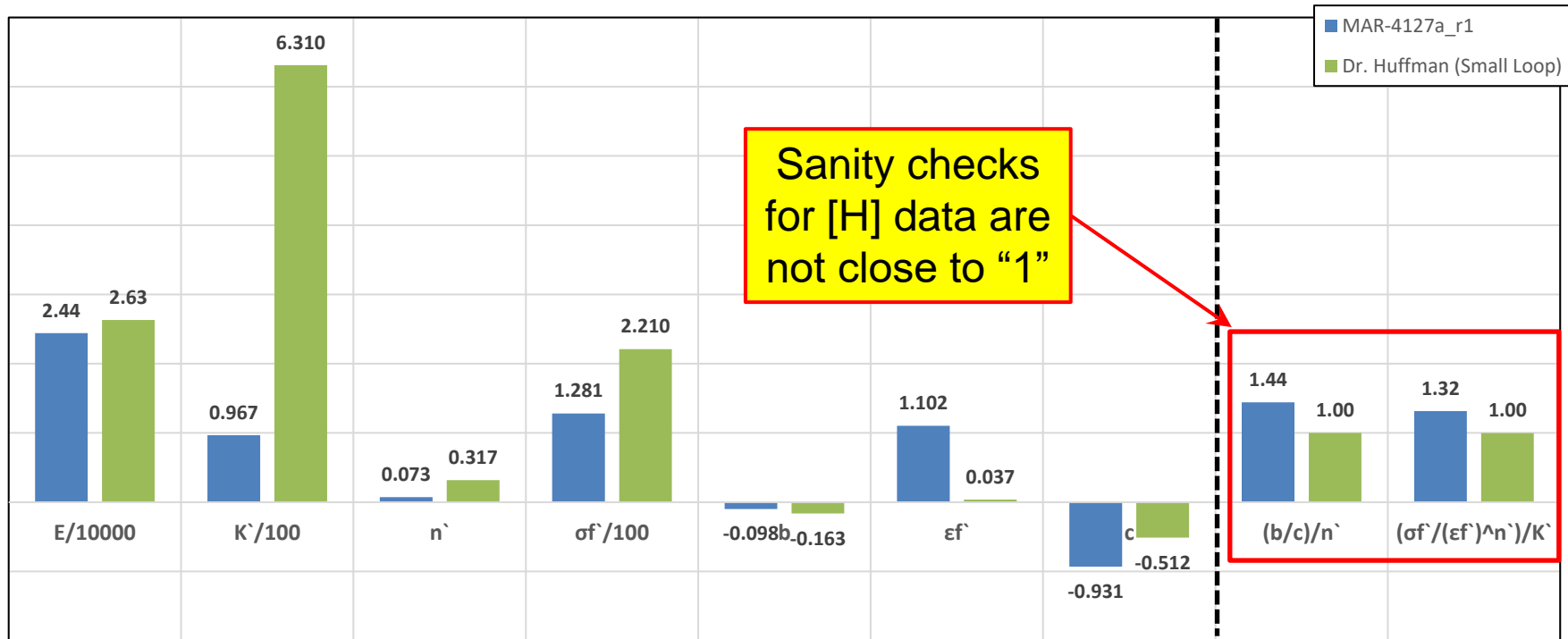
Parameter Comparison



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H

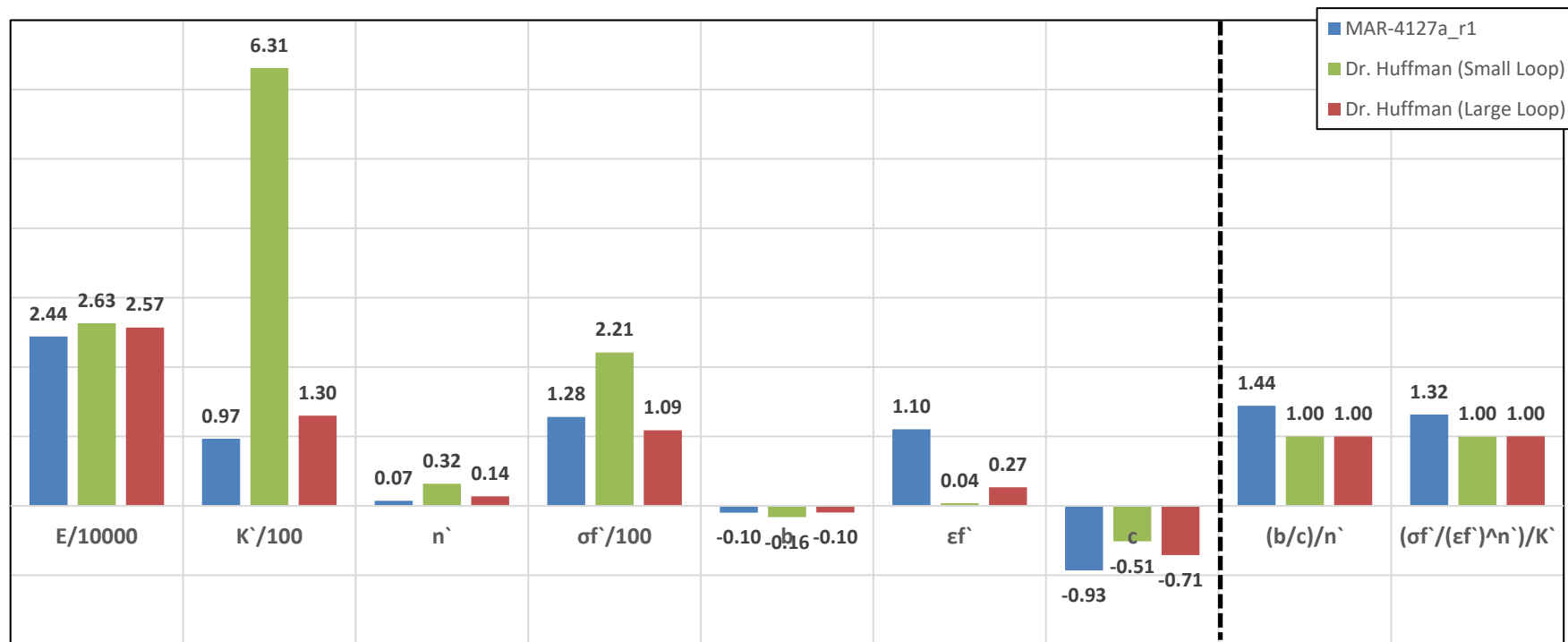
MAR-4127aR1 – D65-45-12...

Parameter Comparison



ε-N Curve Prediction (Both Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

Parameter Comparison



ϵ -N Curve Prediction (Small Hyst. Loop): Dr. H MAR-4127aR1 – D65-45-12...

■ Conclusions...

- To test the method, MAR data was provided to Dr. Huffman...
 - ◆ MAR-4127a r1 – D65-45-12
 - ◆ Small hysteresis loop and corresponding $2N_f$.
- The predicted curve...
 - ◆ Was a poor match (~61%) over parts of the strain range.
 - ◆ Matched reasonably (~31%) within the normal range of [H] testing.
 - ◆ Was not within the 95% Confidence Interval over parts of the strain range.
- The Predicted Parameters...
 - ◆ Showed significant variation.
 - ◆ The [H] data sanity checks were not close to one.

ϵ -N Curve Prediction (Hyst. Loops): Dr. Peter Huffman - Conclusions...

- The predicted curves, generated from a single high strain test, matched the final [H] test data very well over the range of interest...
 - Within ~30% while the 95% Confidence Interval was much larger.
- The Predicted Parameters...
 - The [H] data sanity checks were sometimes close to one.
 - [H] fitting method?
- Overall, the results look promising.



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